15 Reciprocity Theorem

A nuclear reaction rate is related to its inverse reaction by the reciprocity theorem. For a reaction $a + A \rightarrow b + B$ it can be written as

$$\frac{p^2_a}{1 + \delta_a} \sigma_{aA} = \frac{p^2_b}{1 + \delta_b} \sigma_{bB} \rightarrow aA$$

(a) Show that for massive particle $a$ and $A$, it follows that the ratio of backward to forward reaction rate is given by

$$\langle \sigma v \rangle_{bB \rightarrow aA} / \langle \sigma v \rangle_{aA \rightarrow bB} = \frac{(2j_a + 1)(2j_A + 1)(1 + \delta_b)}{(2j_b + 1)(2j_B + 1)(1 + \delta_a)} e^{-Q_{aA \rightarrow bB}/kT}$$

(2)

where $Q_{aA \rightarrow bB}$ is the Q-value of the forward direction of the reaction, which can be written using the kinetic energies as $Q_{aA \rightarrow bB} = E_{bB} - E_{aA}$.

(b) * Now consider a photo-disintegration reaction $A + \gamma \rightarrow b + B$. The photodisintegration decay constant is given by the expression

$$\lambda_{\gamma} = \frac{8\pi}{h^2} \int_{E_t}^{\infty} E^2_{\gamma} e^{E_{\gamma}/kT - 1} \sigma(E_{\gamma}) dE_{\gamma}$$

(3)

where $E_t = Q_{bB \rightarrow a\gamma}$ is the threshold energy for the reaction (most photodisintegration reactions are endothermic) and $E_{\gamma}$ is the photon energy. Show that in this case, the ratio of the reaction rates can be written as

$$\frac{\lambda_{\gamma}}{\langle \sigma v \rangle_{bB \rightarrow aA}} = \left( \frac{2\pi}{h^2} \right)^{3/2} \left( m_{bB} kT \right)^{3/2} \frac{E^2_{\gamma}}{E_t} \frac{(2j_b + 1)(2j_B + 1)}{(2j_A + 1)(1 + \delta_b)} e^{-Q_{bB \rightarrow a\gamma}/kT}$$

(4)

Hint: You can make the assumption that $e^{E_{\gamma}/kT - 1} \approx e^{E_{\gamma}/kT}$ for $E_{\gamma} >> kT$.

16 Abundance evolution of $^{25}\text{Al}$

There are several processes that can influence the abundance of an isotope $X$ in a star over time. In general, we can write

$$\frac{d(N_X)}{dt} = \sum_i \pm r_i$$

(5)

where we have to sum (+) over all production mechanisms and subtract all destruction mechanisms (-) with individual reaction rates $r_i$.

(a) Find possible production and destruction mechanisms $^{25}\text{Al}$ using the section of the nuclide chart below.

![Nuclide chart image]
(b) Write down the explicit expression for the time evolution of the $^{25}\text{Al}$ abundance $\frac{dN(^{25}\text{Al})}{dt}$ using your result from (a).

(c) Now we consider two specific reactions that can destroy the nucleus $^{25}\text{Al}$: the capture reaction $^{25}\text{Al}(p,\gamma)^{26}\text{Si}$ and $\beta^+$-decay. Neglecting other processes, determine the dominant destruction process at a stellar temperature of $T = 0.3$ GK assuming a proton capture rate of $N_A \langle \sigma v \rangle = 1.8 \cdot 10^{-3}$ cm$^3$ mol$^{-1}$ s$^{-1}$ and give the total lifetime of $^{25}\text{Al}$ with respect to both processes. Assume a stellar density of $\rho = 10^4$ g/cm$^3$ and a hydrogen mass fraction of $X_H = 0.7$.

(d) Calculate at which temperature $T$ the two processes would contribute equally to the destruction of $^{25}\text{Al}$. To do this, assume that the beta decay rate is independent of $T$. Further, assume a non-resonant proton capture reaction rate to calculate the reaction’s power law dependence index $n$ first.